

Math 410

Exam 2

Due July 29, 2019

Name:

1. Show that, for any $m \times n$ matrix A , the solution set to the homogeneous system $A\vec{x} = \vec{0}$ is a subspace of $\mathbb{R}^n$ and the set $\{\vec{b} \mid A\vec{x} = \vec{b} \text{ is consistent}\}$ is a subset of $\mathbb{R}^m$. Is the set of solutions to a non-homogeneous system $A\vec{x} = \vec{b}$ a subspace of $\mathbb{R}^n$? Explain why or why not.
2. Let V be a vector space and let W_1 and W_2 be subspaces. Show that the intersection $W_1 \cap W_2$ is also a subspace of V . Is it true that the union $W_1 \cup W_2$ is also a subspace of V ? If so, prove it. If not, provide a counterexample. For 5 points extra credit, construct the "smallest" subspace of V containing $W_1 \cup W_2$.
3. Find a subset of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$ that forms a basis for $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$ and write the remaining vectors as a linear combination of those where

$$\begin{aligned}\vec{v}_1 &= (2, -1, 4), \quad \vec{v}_2 = (1, 4, -2), \quad \vec{v}_3 = (3, -6, 10), \\ \vec{v}_4 &= (-1, 5, -6), \quad \vec{v}_5 = (4, 7, 0)\end{aligned}$$

4. Find the transition matrix $P_{B \rightarrow B'}$ from the basis B to the basis B' where $B = \{(-1, 1, 2), (1, -1, 1), (0, 1, 1)\}$ and $B' = \{(2, 1, 1), (2, 0, 1), (1, 1, -1)\}$. Then apply the matrix to find $(\vec{v})_{B'}$ where $(\vec{v})_B = (-3, 2, 9)$.